Adjusted Greeks, Point Types and Skew Dimensions

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The adjusted greeks capture the effect of the change in volatility resulting from the skew curve sliding along the volatility path (or slide). There are two cases in which the greeks - specifically, the delta, gamma, zeta and epsilon - are adjusted:

- When *Pivot Type* is set to *Floating Skew*.
- When *Pivot Type* is set to *Fixed Skew*, *Apply point type to center* is set to *Yes*, and the *Point type* has a relative center. In this special case, a volatility slide is assumed with the slope and curvature both being equal to zero.

The adjusted risk greeks are calculated from the formulas below. Each includes a raw or unadjusted term (i.e., that ignores any change in volatility) and an adjustment term. The formulas follow from the definition of the respective greek by applying the chain rule to include the effect of the change in volatility. The slope and curvature that are included in these formulas are defined to be the first and second derivatives, respectively, of the volatility skew at the given strike.

$$\delta = \delta_0 + vega \cdot slope$$

$$\gamma = \gamma_0 + 2 \cdot \zeta \cdot slope + volga \cdot slope^2 + vega \cdot curvature$$

$$\epsilon = \varepsilon_0 + vegaTheta \cdot slope$$

$$\zeta = \zeta_0 + volga \cdot slope$$

These first and second derivatives are calculated by bumping the spot price up and down an infinitesimal amount and calculating the resulting change in volatility at that strike.

Volatility calculations in Metro involve the following three skew curve parameters which are all set in *Theoretical Model Wizard*. Specifically:

1. *Skew Type* – This defines the form of the volatility function – e.g., 2-sided, spline, etc.
2. *Point Type* – These are the units in which the center and wings are defined. There are currently the eight Point Types of Strike, Strike Offset, Standard Deviation, Log Simple, Time Moneyness, Standard Moneyness, Delta, and Symmetric Delta.
3. *Skew Dimension* – These are the units into which a given strike is converted for calculating the volatility at that strike. Currently, there are eight Skew Dimensions in Metro. As a note, the parameters of any given *Skew Type* (e.g., the slope or curvature for a 2-sided curve) are in units of the *Skew Dimension*. In contrast, the volatility slide parameters are always in units of *strike*.

Also, the definition of Point Type and Skew Dimension will in general depend on which of the three types of pricing models is being used: *Lognormal*, *Normal* or *CEV*. The *Lognormal* pricing model will herein be considered the base model for which the formulas are presented. Any alternate formulas for either the *Normal* or the *CEV* pricing models will be subsequently presented.

After a volatility slide, the volatility at any given strike, $\sum(K, C)$, where *K* and *C* are the strike and final center strike, respectively, can be expressed as the sum of two components:

$$\sum(K, C) = \sigma(C_0, C) + g[f(K) - f(C)]$$

where

$$\sigma(C_0, C) = \sigma_0 + slideS \cdot (C - C_0) + \frac{1}{2} \cdot slideC \cdot (C - C_0)^2$$

In the above equation, σ_0 and $\sigma(C_0, C)$ are the initial and final center volatilities, C_0 and C are the initial and final center strikes, and *slideS* and *slideC* are the slope and curvature, respectively, of the volatility slide. In the last term of the first equation, *g[]* and *f()* represent the *Skew Type* function and the *Skew Dimension* function, respectively.

It is the center strike that slides along the volatility path as the underlying price moves. If the Point Type is not a function of the center strike or the center volatility, the shape of the skew curve will remain unchanged as the center strike slides along the volatility path.

As a note, prior to Metro Release 6.1.6, the center of the skew curve for point types that now have the forward price as the default used to have the spot price as the default.

Lognormal Pricing Model: Formulas

Below is a list of the *Point Types* that includes their definition and default center, both old and new, for the *Lognormal* pricing model. Here, the parameter d_1 has the same definition as per the usual Black-Scholes pricing formula and σ is the center volatility which is a *Lognormal* (as opposed to *Normal*) volatility.

Pricing model: Lognormal				
	definition		**default centerStrike**	
Point Type	**old**	**new**	**old**	**new**
Strike	K	no change	NA	NA
Strike Offset	$K - S$	$K - F$	S	F
Standard Deviation	$\frac{(K-S)}{S\sigma\sqrt{t}}$	$\frac{(K-F)}{S\sigma\sqrt{t}}$	S	F
Log Simple	$ln(K/F)$	no change	F	no change
Time Moneyness	$\frac{ln(K/F)}{\sqrt{t}}$	no change	F	no change
Standard Moneyness	$\frac{ln(K/F)}{\sigma\sqrt{t}}$	no change	F	no change
Delta	$e^{-qt}N(d_1)$	no change	$Fe^{\frac{\sigma^2 t}{2} - \sigma\sqrt{t}N^{-1}(e^{qt}/2)}$	no change
Symmetric Delta	$N(d_1)$	no change	$Fe^{\frac{\sigma^2 t}{2}}$	no change

In the definition of *Delta*, *q* is the dividend rate (or equivalently (*r – b)*) and $N()^{-1}$is the inverse normal cumulative distribution. The default values for *Delta* and *Symmetric Delta* are both 0.5 which are then inverted to arrive at the formulas for *default centerStrike* above.

Below is a list of the *Skew Dimensions* for the *Lognormal* pricing model along with their definitions.

Pricing model: Lognormal	
Skew Dimension	**definition**
Strike	K
Relative Strike	K/F
Symmetric Delta	$1 - N(d_1)$
Standard Deviation	$\frac{(K-F)}{S\sigma\sqrt{t}}$
Log Simple Moneyness	$ln(K/F)$
Time-Weighted Moneyness	$\frac{ln(K/F)}{\sqrt{t}}$
Vol-Weighted Moneyness	$\frac{ln(K/F)}{\sigma}$
Standard Moneyness	$\frac{ln(K/F)}{\sigma\sqrt{t}}$

Normal Pricing Model: Formulas

The *Lognormal* pricing model definitions for *Point Types* and *Skew Dimensions* apply to the *Normal* pricing model as well except as noted below. It is important to note that in all formulas σ is the relevant centerVol for the pricing model – i.e., it is the volatility that gets passed to the pricing model. (In general, to convert a *Normal* vol to a *Lognormal* vol, one needs to divide by the spot price, *S*.)

Pricing model: Normal				
	definition		**default centerStrike**	
Point Type	**old**	**new**	**old**	**new**
Standard Deviation	$\frac{(K-S)}{\sigma\sqrt{t}}$	$\frac{(K-F)}{\sigma\sqrt{t}}$	S	F
Standard Moneyness	$\frac{ln(K/F)}{\sigma/S\sqrt{t}}$	no change	F	no change
Delta	$e^{-qt}N(d)$	no change	$Fe^{\frac{\sigma^2 t}{2} - \sigma\sqrt{t}N^{-1}(e^{qt}/2)}$	no change

A note for the above table:

- The *Delta* is a *Normal* (pricing model) delta and the parameter *d* is defined as per the *Normal* pricing model – i.e., $d = (F - K)/(\sigma\sqrt{t})$.

Below is a list of the *Skew Dimensions* for the *Normal* pricing model. Only those for which the definition differs from the *Lognormal* model are included.

Pricing model: Normal	
Skew Dimension	**definition**
Standard Deviation	$\frac{(K-F)}{\sigma\sqrt{t}}$
Standard Moneyness	$\frac{ln(K/F)}{\sigma/S\sqrt{t}}$

CEV Pricing Model: Formulas

The *Lognormal* pricing model definitions for *Point Types and Skew Dimensions* apply to the *CEV* pricing model as well except as noted below. The *CEV* pricing model includes two additional parameters: 1) *maxPrice* (*M*), which is the maximum allowed price for the futures, and 2) α, which controls the scaling of the volatility parameter ($\alpha = 0$ corresponds to the *Normal* pricing model).

Pricing model: CEV				
	definition		**default centerStrike**	
Point Type	**old**	**new**	**old**	**new**
Standard Deviation	$\frac{(K-S)}{\sigma\sqrt{t}} \cdot \frac{F}{(M-F)^{\alpha}}$	$\frac{(K-F)}{\sigma\sqrt{t}} \cdot \frac{F}{(M-F)^{\alpha}}$	S	F
Standard Moneyness	$\frac{ln(K/F)}{\sigma\sqrt{t}} \cdot \frac{1}{(M-F)^{\alpha}}$	no change	F	no change
Delta	$no\ analytic\ formula$	no change	no change	no change

A note for the above table:

- The *Delta* is a *CEV* (pricing model) delta and since the model used is not analytical, the strike corresponding to a *Delta* of 0.5 must be determined through iteration.

Below is a list of the *Skew Dimension* definitions for the *CEV* pricing model. Only those for which the definition differs from the *Lognormal* model are included.

Pricing model: CEV	
Skew Dimension	**definition**
Standard Deviation	$\frac{(K-F)}{\sigma\sqrt{t}} \cdot \frac{F}{(M-F)^{\alpha}}$

Standard Moneyness	$\frac{ln(K/F)}{\sigma\sqrt{t}} \cdot \frac{1}{(M-F)^{\alpha}}$